GLOBAL ASTRONOMY & ASTROPHYSICS CHALLENGE

ROUND 2

A Preview of the Question System

10 Free-Response Questions with Full Worked Solutions

TOPICS COVERED

Spherical Astronomy · Stellar Physics · Cosmology
Exoplanet Science · Radioactive Decay · Stellar Motion

2026

Reference Sheet

Physical and Astronomical Constants

Quantity	Symbol	Value
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Solar mass	$M_\odot$	$1.989 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_\odot$	$6.957 \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_\odot$	$3.828 \times 10^{26}\,\mathrm{W}$
Solar effective temperature	$T_\odot$	$5772\,\mathrm{K}$
Astronomical unit	AU	$1.496 \times 10^{11}\,\mathrm{m}$
Parsec	pc	$3.086 \times 10^{16}\,\mathrm{m}$
Stefan–Boltzmann constant	σ	$5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Wien's displacement constant	b	$2.898 \times 10^{-3}\,\mathrm{m\,K}$

Useful Relations

Newtonian Gravity

$$F_g = \frac{GMm}{r^2}$$

Altitude Relation

$$\sin h = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H$$

Stefan–Boltzmann Law

$$F = \sigma T^4$$

Circular Orbital Speed

$$v = \frac{2\pi r}{P}$$

These fundamental relations underpin the solutions; any specialised formula needed is derived from them within the solution itself.

Questions

Q1

At an observatory located at latitude 30°N, an astronomer observes a star whose declination is +20°. At the moment of observation, the star is exactly 3 hours of right ascension east of the local meridian.

What is the star's altitude above the horizon, in degrees?

SOLUTION

The hour angle is

$$H = 3 \text{ hours} \times 15^\circ/\text{hour} = 45^\circ.$$

Using the spherical altitude relation

$$\sin h = \sin\phi \sin\delta + \cos\phi \cos\delta \cos H,$$

where $\phi = 30^\circ$, $\delta = 20^\circ$, and $H = 45^\circ$:

$$\sin h = \sin 30^\circ \sin 20^\circ + \cos 30^\circ \cos 20^\circ \cos 45^\circ$$

$$\sin h = (0.5000)(0.3420) + (0.8660)(0.9397)(0.7071)$$

$$\sin h = 0.1710 + 0.5754 = 0.7464$$

$$\boxed{h = \arcsin(0.7464) \approx 48.3^\circ}$$

Q2

An old stellar catalogue lists a nearby star with a parallax of 25 milliarcseconds. Modern observations give its proper motion as 0.80 arcseconds per year.

What is the star's tangential velocity in km/s?

SOLUTION

The parallax gives the distance:

$$d = \frac{1}{p} = \frac{1}{0.025} = 40 \text{ pc}.$$

The tangential velocity follows from $v_t = \omega d$, where the angular speed is

$$\omega = \mu \left(\frac{\pi}{180 \times 3600} \right) \text{ rad/yr}.$$

Converting pc to km and yr to seconds,

$$v_t = \mu\, d\, \frac{4.848 \times 10^{-6} \times 3.086 \times 10^{13}}{3.156 \times 10^7} \text{ km/s},$$

where the constant combination evaluates to 4.74, so

$$v_t = 4.74\,\mu\,d.$$

Substituting $\mu = 0.80$ arcsec/yr and $d = 40$ pc:

$$v_t = 4.74(0.80)(40)$$

$$\boxed{v_t \approx 151.7 \text{ km/s}}$$

Q3

During a meteor observing campaign, a camera records a luminous trail that appears to cover 18.0° of the sky. The meteor is approximately 90 km from the camera during the recorded part of the trail, and the trail lasts 0.42 seconds. Assume the meteor's path is perpendicular to the observer's line of sight over the recorded interval.

What was its speed in km/s?

SOLUTION

Convert the angular size to radians:

$$\theta = 18.0^\circ \times \frac{\pi}{180^\circ} \approx 0.3142 \text{ rad}.$$

The length of the trail at a distance $r = 90$ km is

$$s = r\theta = (90)(0.3142) \approx 28.27 \text{ km}.$$

The speed is the distance divided by the duration $t = 0.42$ s, so

$$v = \frac{s}{t} = \frac{28.27}{0.42}$$

$$\boxed{v \approx 67.3 \text{ km/s}}$$

Q4

A newly catalogued star has an effective temperature of 8000 K and a radius 2.5 times that of the Sun.

What is its luminosity in units of the solar luminosity?

SOLUTION

The Stefan–Boltzmann law gives the energy flux per unit surface area,

$$F = \sigma T^4.$$

The luminosity follows by multiplying the flux by the star's surface area,

$$L = 4\pi R^2 F = 4\pi R^2 \sigma T^4.$$

Applying this to the star and to the Sun, then dividing the two relations,

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4,$$

where $R/R_\odot = 2.5$ and $T/T_\odot = 8000/5772$. Substituting these values:

$$\frac{L}{L_\odot} = (2.5)^2 \left(\frac{8000}{5772}\right)^4$$

$$\frac{L}{L_\odot} = 6.25\,(1.386)^4$$

$$\frac{L}{L_\odot} = 6.25(3.690)$$

$$\boxed{\frac{L}{L_\odot} \approx 23.1}$$

Q5

A spectrograph is being used to study a white dwarf. Its mass is $0.90\,M_\odot$ and its radius is $0.0090\,R_\odot$. The gravitational redshift of the star is small enough that

$$z \approx \frac{GM}{Rc^2}.$$

Astronomers often express this redshift as an equivalent velocity using $v = zc$.

What is this equivalent velocity in km/s?

SOLUTION

The equivalent velocity is

$$v = zc = \frac{GM}{Rc^2}c = \frac{GM}{Rc}.$$

Converting the mass and radius to SI units,

$$M = 0.90 M_\odot = 0.90(1.989 \times 10^{30}) = 1.790 \times 10^{30}\ \text{kg},$$

$$R = 0.0090 R_\odot = 0.0090(6.957 \times 10^{8}) = 6.26 \times 10^{6}\ \text{m}.$$

So

$$v = \frac{(6.674 \times 10^{-11})(1.790 \times 10^{30})}{(6.26 \times 10^{6})(3.00 \times 10^{8})}$$

$$v = \frac{1.195 \times 10^{20}}{1.878 \times 10^{15}} \approx 6.36 \times 10^{4}\ \text{m/s}$$

$$\boxed{v \approx 63.6\ \text{km/s}}$$

Q6

Two stars are observed to orbit one another. Their angular semimajor axis is measured to be 0.120 arcseconds, and the system is 25.0 pc away. The orbital period is 1.50 years. Assuming the measured angular semimajor axis corresponds to the semimajor axis of the relative orbit:

What is the total mass of the system in solar masses?

SOLUTION

Convert the angular semimajor axis to a physical size using the small-angle formula:

$$a = \alpha\, d = (0.120)(25.0) = 3.00 \text{ AU}.$$

For a circular relative orbit the gravitational force supplies the centripetal force,

$$\frac{GMm}{a^2} = \frac{mv^2}{a},$$

which gives

$$M = \frac{v^2 a}{G}.$$

With the circular orbital speed $v = 2\pi a/P$,

$$M = \frac{4\pi^2 a^3}{GP^2}.$$

In solar-system units (a in AU, P in years, M in solar masses) this is Kepler's third law,

$$M = \frac{a^3}{P^2}.$$

Substituting $a = 3.00$ AU and $P = 1.50$ yr:

$$M = \frac{(3.00)^3}{(1.50)^2} = \frac{27.0}{2.25}$$

$$\boxed{M = 12.0\, M_\odot}$$

Q7

In an investigation of a supernova remnant, astronomers model one radioactive species produced in the explosion. Its half-life is 6.1 days.

Exactly 30 days after the explosion, what percentage of the original nuclei remain?

SOLUTION

Thirty days correspond to

$$\frac{30}{6.1} \approx 4.918$$

half-lives. The remaining fraction is

$$\left(\frac{1}{2}\right)^{4.918}.$$

As a percentage,

$$100\left(\frac{1}{2}\right)^{30/6.1}$$

$$\boxed{\approx 3.13\%}$$

Q8

A radio telescope detects a relic background signal whose wavelength today is 1.90 mm. The radiation was emitted when the universe had a temperature of approximately 3000 K. Assume that at emission the wavelength corresponding to the peak of the spectrum can be estimated using Wien's law.

What was the redshift of the radiation?

SOLUTION

The emitted peak wavelength follows from Wien's law,

$$\lambda_{\text{em}} = \frac{b}{T} = \frac{2.898 \times 10^{-3}}{3000}$$

$$\lambda_{\text{em}} \approx 9.66 \times 10^{-7} \text{ m}.$$

The redshift is defined by

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}},$$

so, substituting $\lambda_{\text{obs}} = 1.90 \times 10^{-3}$ m,

$$z = \frac{1.90 \times 10^{-3}}{9.66 \times 10^{-7}} - 1$$

$$z = 1966.9 - 1$$

$$\boxed{z \approx 1966}$$

Q9

Adel and Samy are analyzing the spectrum of a rapidly rotating star. They notice that two spectral lines which should have rest wavelengths of 500.000 nm and 600.000 nm are measured at different wavelengths on opposite sides of the star. For the first line, the two measured wavelengths are 499.200 nm and 500.800 nm. For the second line, the corresponding measurements are 599.040 nm and 600.960 nm. Assume the shifts are entirely due to the line-of-sight rotational motion and that the star's equatorial speed is the same on both sides.

What is the rotational speed of the star in km/s?

SOLUTION

The Doppler shift relates wavelength change to velocity:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}.$$

For the 500 nm line,

$$\Delta\lambda = 500.800 - 500.000 = 0.800 \text{ nm},$$

so

$$\frac{v}{c} = \frac{0.800}{500.000}.$$

Substituting the measured shift,

$$v = (3.00 \times 10^5)\left(\frac{0.800}{500.000}\right)$$

$$v = (3.00 \times 10^5)(1.60 \times 10^{-3})$$

$$\boxed{v = 480 \text{ km/s}}$$

Check with the second line:

$$\Delta\lambda = 600.960 - 600.000 = 0.960 \text{ nm}, \qquad \frac{0.960}{600.000} = 1.60 \times 10^{-3},$$

which gives the same rotational speed of 480 km/s.

Q10

A planet is discovered around a Sun-like star. During transit, the star's brightness decreases by exactly 1.00%. The star has a radius of 0.90 $R_\odot$. Radial-velocity observations give the planet an orbital velocity semi-amplitude of 70 m/s, and the orbital period is 3.20 days. Assume the orbit is circular and edge-on, and that the planet's mass is much smaller than the star's mass.

What is the planet's mean density in g/cm^3?

SOLUTION

The transit depth is the fraction of the stellar disk blocked by the planet,

$$\frac{\Delta F}{F} = \frac{\pi R_p^2}{\pi R_*^2} = \left(\frac{R_p}{R_*}\right)^2 = 0.0100.$$

So

$$\frac{R_p}{R_*} = 0.10.$$

Because $R_* = 0.90 R_\odot$,

$$R_p = 0.090 R_\odot = 0.090(6.957 \times 10^8) = 6.26 \times 10^7 \text{ m}.$$

The radial-velocity amplitude follows from Newtonian gravity. For a circular orbit the planet's speed is

$$v = \sqrt{\frac{GM_*}{a}} = \frac{2\pi a}{P},$$

and the star's reflex velocity is reduced by the mass ratio,

$$K = \frac{M_p}{M_*} v = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p}{M_*^{2/3}},$$

where $M_p \ll M_*$. Rearranging for the planet's mass,

$$M_p = K\, M_*^{2/3} \left(\frac{P}{2\pi G}\right)^{1/3}.$$

Substituting $K = 70$ m/s, $P = 3.20$ days $= 2.7648 \times 10^5$ s and $M_* = 1\, M_\odot = 1.989 \times 10^{30}$ kg,

$$M_p = 70\,(1.989 \times 10^{30})^{2/3} \left(\frac{2.7648 \times 10^5}{2\pi(6.674 \times 10^{-11})}\right)^{1/3}$$

$$M_p \approx 9.63 \times 10^{26} \text{ kg} = 4.84 \times 10^{-4} M_\odot.$$

The mean density is

$$\rho = \frac{M_p}{\frac{4}{3}\pi R_p^3} = \frac{9.63 \times 10^{26}}{\frac{4}{3}\pi(6.26 \times 10^7)^3}$$

$$\rho \approx 937 \text{ kg/m}^3.$$

As $1 \text{ g/cm}^3 = 10^3 \text{ kg/m}^3$,

$$\boxed{\rho \approx 0.937 \text{ g/cm}^3}$$