GLOBAL ASTRONOMY & ASTROPHYSICS CHALLENGE

ROUND 2

Knockout Bank: Semi-Finals

10 Free-Response Questions with Full Worked Solutions

TOPICS COVERED

Observational Astronomy · Orbital Mechanics · Astrophysics
Planetology · Compact Objects · Cosmology

2026

Reference Sheet

Physical and Astronomical Constants

Quantity	Symbol	Value
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Solar mass	$M_\odot$	$1.989 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_\odot$	$6.957 \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_\odot$	$3.828 \times 10^{26}\,\mathrm{W}$
Solar effective temperature	$T_\odot$	$5772\,\mathrm{K}$
Astronomical unit	AU	$1.496 \times 10^{11}\,\mathrm{m}$
Parsec	pc	$3.086 \times 10^{16}\,\mathrm{m}$
Stefan–Boltzmann constant	σ	$5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Wien's displacement constant	b	$2.898 \times 10^{-3}\,\mathrm{m\,K}$
Reduced Planck constant	$\hbar$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Electron mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg}$
Earth mass	$M_\oplus$	$5.972 \times 10^{24}\,\mathrm{kg}$
Earth radius	$R_\oplus$	$6.371 \times 10^{6}\,\mathrm{m}$
Hubble constant	H_0	$70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$
Megaparsec	1 Mpc	$3.086 \times 10^{19}\,\mathrm{km}$
Arcseconds per radian		206 265
CMB temperature today	T_0	$2.725\,\mathrm{K}$
Sun main-sequence lifetime		10 Gyr
Age of the universe today	t_0	13.8 Gyr
Year	1 yr	$3.156 \times 10^{7}\,\mathrm{s}$
Thomson cross-section	σ_T	$6.65 \times 10^{-29}\,\mathrm{m^2}$
Proton mass	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$

Useful Relations

Gravitation

$$F = \frac{GMm}{r^2}$$

Circular Orbital Speed

$$v^2 = \frac{GM}{r}$$

Orbital Energy (Vis-Viva)

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

Wien Displacement

$$\lambda_{\max} T = b$$

Radial Doppler Shift

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v_r}{c}$$

Flux and Luminosity

$$F = \frac{L}{4\pi d^2}$$

These six master equations seed every derivation in this booklet. Each question asks for a specialised quantity; obtaining it from them always requires a short chain of reasoning. None of the answers can be reached by substituting numbers into the master equations directly.

Questions

Stage 2: The Semi-Finals

The Library of Black Glass, where old myths conceal measurements and every locked room is a scientific argument.

Question 11: The Photograph with No Distance (Obs Ast)

Rayan and Mara are now invited to the underground Library of Black Glass, where every record was deliberately stripped of its distance. One photograph shows a star labeled only "Sable One." Its apparent magnitude is 8.40 and its absolute magnitude is 1.40. The intervening dust lets only 47.9% of the star's light reach the telescope in the observed band. The librarian says the missing distance is encoded in the magnitude difference, not in a scale bar.
Infer the distance to Sable One in parsecs.

SOLUTION

The dusty screen transmits 47.9% of the light, so its dimming in magnitudes is

$$A = 2.5\log_{10}\frac{1}{0.479} \approx 0.80$$

The distance modulus with extinction is

$$m - M = 5\log_{10}(d) - 5 + A$$

$$8.40 - 1.40 = 5\log_{10}(d) - 5 + 0.80$$

$$6.20 = 5\log_{10}(d) - 5$$

$$5\log_{10}(d) = 11.20$$

$$\log_{10}(d) = 2.24$$

$$d = 10^{2.24}$$

$$\boxed{d \approx 174\,\mathrm{pc}}$$

Question 12: The Blue Line of the Prisoner (Obs Ast)

The spectrograph in the Library of Black Glass records a hydrogen line whose laboratory wavelength is 656.28 nm. During the observation, the instrument reports 656.61 nm. A

calibration note says that the detector adds 0.09 nm to every wavelength it reports.
At that instant the observatory is moving directly away from the star at $18.0\,\mathrm{km\,s^{-1}}$. Treat all velocities as non-relativistic. Rayan is interested in the star's motion itself, measured in the reference frame of the surrounding stellar neighborhood.
Separate the star's own motion: what is its radial recession velocity in $\mathrm{km\,s^{-1}}$?

SOLUTION

Correct the detector reading first.

$$\lambda_{\text{true}} = 656.61 - 0.09 = 656.52\,\mathrm{nm}$$

The fractional change in wavelength of a non-relativistically moving source equals the corresponding fractional change in light speed components along the line of sight. Thus

$$\frac{\lambda_{\text{true}} - \lambda_0}{\lambda_0} = \frac{v_{\text{rel}}}{c}$$

$$\frac{656.52 - 656.28}{656.28} = 3.657 \times 10^{-4}$$

$$v_{\text{rel}} = (3.00 \times 10^5)(3.657 \times 10^{-4})$$

$$v_{\text{rel}} \approx 109.7\,\mathrm{km\,s^{-1}}$$

This is the recession speed between the observatory and the star. Because the observatory itself is moving away from the star at $18.0\,\mathrm{km\,s^{-1}}$, the star's recession relative to the surrounding neighborhood is smaller by that amount.

$$v_\star = 109.7 - 18.0$$

$$\boxed{v_\star \approx 91.7\,\mathrm{km\,s^{-1}}}$$

Question 13: The Road Between Worlds (Obs Ast: Orbital Mechanics)

Beyond the library is a launch chamber containing a miniature spacecraft called Persephone. The craft begins in a circular orbit at 1.00 AU around a Sun-like star and must reach a circular orbit at 2.50 AU. The engineer who designed the chamber insists on a two-impulse Hohmann transfer and ignores all propulsion losses. Rayan is asked for the total ideal change in speed, counting both impulses.
Determine the total Δv required for the transfer in $\mathrm{km\,s^{-1}}$.

SOLUTION

The initial and final orbital radii are

$$r_1 = 1.00\,\mathrm{AU}, \qquad r_2 = 2.50\,\mathrm{AU}$$

The transfer ellipse has semi-major axis

$$a_t = \frac{r_1 + r_2}{2} = 1.75\,\mathrm{AU}$$

The circular speed at r_1 is

$$v_1 = \sqrt{\frac{\mu}{r_1}} \approx 29.79\,\mathrm{km\,s^{-1}}$$

The speed at the first burn point is

$$v_{t1} = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_t}\right)} \approx 35.60\,\mathrm{km\,s^{-1}}$$

$$\Delta v_1 = v_{t1} - v_1 \approx 5.82\,\mathrm{km\,s^{-1}}$$

At the outer burn point

$$v_{t2} = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_t}\right)} \approx 14.24\,\mathrm{km\,s^{-1}}$$

$$v_2 = \sqrt{\frac{\mu}{r_2}} \approx 18.84\,\mathrm{km\,s^{-1}}$$

$$\Delta v_2 = v_2 - v_{t2} \approx 4.60\,\mathrm{km\,s^{-1}}$$

$$\Delta v_{\text{total}} = \Delta v_1 + \Delta v_2$$

$$\boxed{\Delta v_{\text{total}} \approx 10.41\,\mathrm{km\,s^{-1}}}$$

Question 14: The White Star's Hidden Weight (Astrophysics)

A statue of a white dwarf stands at the center of the library. Its surface is dense enough that even the wavelength of a spectral line carries information about the star's gravity. The dwarf has mass $1.00M_\odot$ and radius $0.0100R_\odot$. A laboratory line at $500\,\mathrm{nm}$ is measured from the surface. The archive recalls that a photon climbing out of the star's well loses the gravitational energy GMm_{ph}/R, where the photon's mass-equivalent is $m_{\text{ph}} = h\nu/c^2$.
Estimate the increase in wavelength, $\Delta\lambda$, in nanometres.

SOLUTION

The photon loses gravitational energy, so its frequency falls. Since $E = h\nu$ and $\nu = c/\lambda$, a fractional energy loss corresponds, in magnitude, to the fractional wavelength increase.

$$\frac{\Delta E}{E} = \frac{GM(h\nu/c^2)}{h\nu R} = \frac{GM}{Rc^2}$$

Thus the gravitational redshift obeys

$$\frac{\Delta\lambda}{\lambda_0} \approx \frac{GM}{Rc^2}$$

With $R = 0.0100 R_\odot$

$$R = 0.0100(6.96 \times 10^8) = 6.96 \times 10^6\,\text{m}$$

$$M = 1.989 \times 10^{30}\,\text{kg}$$

$$z = \frac{(6.674 \times 10^{-11})(1.989 \times 10^{30})}{(6.96 \times 10^6)(3.00 \times 10^8)^2}$$

$$z \approx 2.119 \times 10^{-4}$$

For small redshift

$$\Delta\lambda = z\lambda_0$$

$$\Delta\lambda = (2.119 \times 10^{-4})(500\,\text{nm})$$

$$\boxed{\Delta\lambda \approx 0.106\,\text{nm}}$$

Question 15: The Furnace with No Flame (Astrophysics)

At the center of the library, Rayan finds a star whose atmosphere has been studied spectroscopically. The star lies 20.0 pc away. Its bolometric flux at Earth is $1.18 \times 10^{-9}\,\text{W}\,\text{m}^{-2}$, while its spectrum peaks at $362.3\,\text{nm}$.
The archive gives the definition of flux as received power per unit area and the blackbody relation for emitted power per unit surface area. It gives no luminosity and no radius.
Extract the star's radius in solar radii.

SOLUTION

The measured flux spreads over spheres centered on the star. If the luminosity is L, the same power passes through a sphere of radius d, so

$$F = \frac{L}{4\pi d^2}$$

Thus

$$L = 4\pi d^2 F$$

The distance is

$$d = 20.0(3.086 \times 10^{16}) = 6.172 \times 10^{17}\,\text{m}$$

Hence

$$L = 4\pi(6.172 \times 10^{17})^2(1.18 \times 10^{-9})$$

$$L \approx 5.65 \times 10^{27}\,\text{W}$$

Next recover the effective temperature from the location of the blackbody peak. The peak condition gives

$$\lambda_{\max} T = b$$

$$T = \frac{2.898 \times 10^{-3}}{362.3 \times 10^{-9}}$$

$$T \approx 7998\,\mathrm{K}$$

The luminosity of a spherical blackbody equals its surface area times its radiated power per unit area. Therefore,

$$L = 4\pi R^2 \sigma T^4$$

Solving for R,

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

$$R \approx 1.392 \times 10^9\,\mathrm{m}$$

Finally,

$$\frac{R}{R_\odot} = \frac{1.392 \times 10^9}{6.957 \times 10^8}$$

$$\boxed{R \approx 2.00 R_\odot}$$

Question 16: The Engine Beneath Hades (Astrophysics)

The deepest room is marked Hades. Inside it, a model accretion disk surrounds a black hole of mass $5.0 \times 10^7 M_\odot$. Ionized hydrogen fills the region above the disk. The only microphysical quantities written on the wall are the Thomson cross-section and the proton mass.
A second panel reports that the disk radiates at exactly the limiting luminosity where radiation can no longer be balanced by gravity. The disk converts 10% of the rest-mass energy of accreted matter into radiation.
Resolve the corresponding mass accretion rate in $\mathrm{kg\,s^{-1}}$.

SOLUTION

Take a small parcel of ionized hydrogen at distance r from the black hole. The radiation carries momentum, so a flux F transfers momentum at a rate proportional to F/c. For Thomson scattering, the force on an electron is

$$F_{\mathrm{rad}} = \frac{F\sigma_T}{c}$$

The luminosity spreads over a sphere, giving

$$F = \frac{L}{4\pi r^2}$$

Thus

$$F_{\text{rad}} = \frac{L\sigma_T}{4\pi r^2 c}$$

The gravitational attraction of the proton is

$$F_{\text{grav}} = \frac{GMm_p}{r^2}$$

At the limiting luminosity, the two forces balance.

$$\frac{L\sigma_T}{4\pi r^2 c} = \frac{GMm_p}{r^2}$$

The distance cancels, leaving

$$L = \frac{4\pi GMm_p c}{\sigma_T}$$

With

$$M = 5.0 \times 10^7(1.989 \times 10^{30}) = 9.945 \times 10^{37}\,\text{kg}$$

we obtain

$$L \approx 6.28 \times 10^{38}\,\text{W}$$

Only 10% of the accreted rest-mass energy appears as radiation, so the radiated power is related to the accretion rate by

$$L = 0.10\,\dot{M}c^2$$

Therefore

$$\dot{M} = \frac{L}{0.10c^2}$$

$$\dot{M} = \frac{6.28 \times 10^{38}}{0.10(3.00 \times 10^8)^2}$$

$$\boxed{\dot{M} \approx 6.98 \times 10^{22}\,\text{kg}\,\text{s}^{-1}}$$

Question 17: The Pulse that Spent a Century (Astrophysics)

Mara finds a pulsar recording named The Heart of Hades. A neutron star is modeled as a uniform solid sphere with mass $1.40M_\odot$ and radius 12 km. At the beginning of a monitored interval, its spin period is 0.050 s. One thousand years later, the period is 0.060 s. The archive treats the lost rotational energy as the total energy carried away during the interval and asks for its average rate of loss.

Establish the average rotational energy loss rate in watts.

SOLUTION

For a uniform sphere

$$I = \frac{2}{5}MR^2$$

$$M = 1.40(1.989 \times 10^{30})\,\text{kg}$$

$$R = 12\,000\,\text{m}$$

$$I \approx 1.604 \times 10^{38}\,\text{kg}\,\text{m}^2$$

The rotational kinetic energy is

$$E_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{2\pi}{P}\right)^2$$

For $P_1 = 0.050\,\text{s}$

$$E_1 \approx 1.266 \times 10^{42}\,\text{J}$$

For $P_2 = 0.060\,\text{s}$

$$E_2 \approx 8.795 \times 10^{41}\,\text{J}$$

The energy lost is

$$\Delta E = E_1 - E_2 \approx 3.87 \times 10^{41}\,\text{J}$$

The elapsed time is approximately $1000(365.25)(24)(3600) = 3.156 \times 10^{10}$ s.

$$P_{\text{loss}} = \frac{\Delta E}{\Delta t}$$

$$\boxed{P_{\text{loss}} \approx 1.23 \times 10^{31}\,\text{W}}$$

Question 18: The Moon that Refused the Dark (Planetology)

A small moon circles a giant planet inside the library's simulation vault. Its surface is black in visible light, but its thermal detector reports the equilibrium temperature. The central planet itself is irrelevant to the heating. The moon orbits a star of luminosity $4L_\odot$ at 0.50 AU and has Bond albedo 0.25. The vault supplies only the blackbody balance condition: the moon absorbs the fraction $(1 - A)$ of the stellar light it intercepts and radiates it back according to the archive's blackbody law for emitted power per unit surface area.
Assess the equilibrium temperature of the moon in kelvin.

SOLUTION

The stellar light reaching the moon spreads over a sphere of radius a, so the received flux is

$$F = \frac{L}{4\pi a^2}$$

The moon intercepts an area πR^2 and absorbs a fraction $(1-A)$. In thermal balance this absorbed power equals the power radiated from the whole surface, $4\pi R^2 \sigma T_{\text{eq}}^4$.

$$\pi R^2 (1-A)\frac{L}{4\pi a^2} = 4\pi R^2 \sigma T_{\text{eq}}^4$$

The radius cancels, leaving

$$T_{\text{eq}}^4 = \frac{L(1-A)}{16\pi\sigma a^2}$$

Here

$$L = 4(3.828 \times 10^{26}) = 1.531 \times 10^{27}\,\mathrm{W}$$

$$a = 0.50(1.496 \times 10^{11}) = 7.480 \times 10^{10}\,\mathrm{m}$$

Thus

$$T_{\text{eq}}^4 = \frac{(1.531 \times 10^{27})(0.75)}{16\pi(5.670 \times 10^{-8})(7.480 \times 10^{10})^2}$$

$$T_{\text{eq}}^4 \approx 7.20 \times 10^{10}\,\mathrm{K}^4$$

$$\boxed{T_{\text{eq}} \approx 518\,\mathrm{K}}$$

Question 19: The Weight of the Empty Universe (Cosmology)

The last room in the library displays a scale marked "critical." The universe in the model contains matter with density parameter $\Omega_m = 0.30$. One display has gone dark, and this quantity is needed to restart it.
Weigh the present matter density in $\mathrm{kg\,m^{-3}}$.

SOLUTION

Convert the Hubble parameter to SI units.

$$H_0 = \frac{70\,\mathrm{km\,s^{-1}}}{3.086 \times 10^{19}\,\mathrm{km}}$$

$$H_0 \approx 2.268 \times 10^{-18}\,\mathrm{s^{-1}}$$

The critical density is

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

$$\rho_c \approx 9.20 \times 10^{-27}\,\mathrm{kg\,m^{-3}}$$

The matter density is

$$\rho_m = 0.30(9.20 \times 10^{-27})$$

$$\boxed{\rho_m \approx 2.76 \times 10^{-27}\,\mathrm{kg\,m^{-3}}}$$

Question 20: The Hourglass at the Edge (Cosmology)

The final black-glass door opens only after Rayan accepts a stranger assignment. The archive describes a universe dominated entirely by a phantom component with constant equation-of-state parameter $w = -1.5$.
For this universe, the archive gives the expansion history

$$H(a) = H_0 a^{-3(1+w)/2}.$$

Rayan is not asked for the present expansion rate. He must determine when the scale factor will diverge. Starting from the definition of the Hubble parameter, $H(a) = \dot{a}/a$, and the identity $da = \dot{a}\,dt$, find the time remaining from today until the Big Rip, in billions of years.

SOLUTION

For $w = -1.5$,

$$1 + w = -0.5$$

and the exponent in the Hubble law is

$$-\frac{3(1+w)}{2} = -\frac{3(-0.5)}{2} = 0.75.$$

Thus

$$H(a) = H_0 a^{3/4}.$$

From $H(a) = \dot{a}/a$ and $da = \dot{a}\,dt$, the elapsed time element is

$$dt = \frac{da}{aH(a)}.$$

The remaining time is obtained by integrating from the present scale factor $a = 1$ to $a \to \infty$.

$$\Delta t = \int_1^\infty \frac{da}{aH_0 a^{3/4}}$$

$$\Delta t = \frac{1}{H_0}\int_1^\infty a^{-7/4}\,da$$

The integral is

$$\int_1^\infty a^{-7/4}\,da = \left[-\frac{4}{3}a^{-3/4}\right]_1^\infty$$

$$\int_1^\infty a^{-7/4}\,da = \frac{4}{3}$$

Therefore

$$\Delta t = \frac{4}{3H_0}$$

Convert the Hubble parameter.

$$H_0 = \frac{70}{3.086 \times 10^{19}}$$

$$H_0 \approx 2.268 \times 10^{-18}\,\mathrm{s}^{-1}$$

Hence

$$\Delta t = \frac{4}{3(2.268 \times 10^{-18})}$$

$$\Delta t \approx 5.878 \times 10^{17}\,\mathrm{s}$$

Convert to billions of years.

$$\Delta t = \frac{5.878 \times 10^{17}}{3.156 \times 10^{16}}$$

$$\boxed{\Delta t \approx 18.6\,\text{billion years}}$$