GLOBAL ASTRONOMY & ASTROPHYSICS CHALLENGE

ROUND 2

Knockout Bank: Best 5th to 8th

10 Free-Response Questions with Full Worked Solutions

TOPICS COVERED

Observational Astronomy · Orbital Mechanics · Astrophysics
Planetology · Compact Objects · Cosmology

2026

Reference Sheet

Physical and Astronomical Constants

Quantity	Symbol	Value
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Solar mass	$M_\odot$	$1.989 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_\odot$	$6.957 \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_\odot$	$3.828 \times 10^{26}\,\mathrm{W}$
Solar effective temperature	$T_\odot$	$5772\,\mathrm{K}$
Astronomical unit	AU	$1.496 \times 10^{11}\,\mathrm{m}$
Parsec	pc	$3.086 \times 10^{16}\,\mathrm{m}$
Stefan–Boltzmann constant	σ	$5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Wien's displacement constant	b	$2.898 \times 10^{-3}\,\mathrm{m\,K}$
Reduced Planck constant	$\hbar$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Electron mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg}$
Earth mass	$M_\oplus$	$5.972 \times 10^{24}\,\mathrm{kg}$
Earth radius	$R_\oplus$	$6.371 \times 10^{6}\,\mathrm{m}$
Hubble constant	H_0	$70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$
Megaparsec	1 Mpc	$3.086 \times 10^{19}\,\mathrm{km}$
Arcseconds per radian		206 265
CMB temperature today	T_0	$2.725\,\mathrm{K}$
Sun main-sequence lifetime		10 Gyr
Age of the universe today	t_0	13.8 Gyr
Year	1 yr	$3.156 \times 10^{7}\,\mathrm{s}$
Thomson cross-section	σ_T	$6.65 \times 10^{-29}\,\mathrm{m^2}$
Proton mass	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$

Useful Relations

Gravitation

$$F = \frac{GMm}{r^2}$$

Circular Speed

$$v = \frac{2\pi r}{P}$$

Orbital Energy

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

Radial Doppler Shift

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v_r}{c}$$

Altitude of a Star

$$\sin h = \sin\phi\,\sin\delta + \cos\phi\,\cos\delta\,\cos H$$

Blackbody Power

$$L = 4\pi R^2 \sigma T^4$$

These six master equations seed every derivation in this booklet. Each question asks for a specialised quantity; obtaining it from them always requires a short chain of reasoning. None of the answers can be reached by substituting numbers into the master equations directly.

Questions

Stage 1: The Best 5th to 8th

A mythic observatory, a missing astronomer, and a sky that does not behave as it should.

Question 1: The North That Moved (Obs Ast)

Rayan was alone in the old hilltop observatory when the dome lights began to flicker. Above the northern horizon, the observatory's mythic projection system had awakened and placed Ursa Major and Ursa Minor into a simulated quarrel. The simulation did something stranger than merely draw the constellations: it shifted the projected position of Polaris southward by 2.5°, as though the Little Bear had been pulled away from the pole. Before shutting the system down, Rayan records that his station is at latitude 30° North and that Polaris originally had declination 88.7°.
Find the new maximum altitude of the simulated Polaris above the northern horizon.

SOLUTION

The new declination is obtained from the stated southward shift.

$$\delta_{\text{new}} = 88.7^\circ - 2.5^\circ$$

$$\delta_{\text{new}} = 86.2^\circ$$

At upper culmination, the altitude of a northern star is

$$h_{\max} = 90^\circ - |\phi - \delta|$$

$$h_{\max} = 90^\circ - |30^\circ - 86.2^\circ|$$

$$h_{\max} = 90^\circ - 56.2^\circ$$

$$\boxed{h_{\max} = 33.8^\circ}$$

Question 2: The Silent Eclipse (Obs Ast)

Three days later, an unexplained light curve arrives from the same archive. A young astronomer named Mara notices that a nearby star dims with almost perfect flat-bottomed timing. The archive labels the event only as "The Lantern Crossing." The star has radius $1.20R_\odot$, and the deepest part of the transit reduces the received flux by 0.81%. The object crossing the star is dark at visible wavelengths. Rayan suspects that the old astronomers hid the object's scale in the geometry of the eclipse rather than in its name.
Work out the radius of the transiting object in Earth radii.

SOLUTION

For a central transit of a dark spherical body, the fractional loss of light is

$$\frac{\Delta F}{F} = \left(\frac{R_p}{R_\star}\right)^2$$

The measured depth is $0.81\% = 0.0081$.

$$\frac{R_p}{R_\star} = \sqrt{0.0081} = 0.09$$

The stellar radius is

$$R_\star = 1.20(109R_\oplus) = 130.8R_\oplus$$

Thus

$$R_p = 0.09(130.8R_\oplus)$$

$$\boxed{R_p = 11.77R_\oplus}$$

Question 3: The Courier at Periapsis (Obs Ast: Orbital Mechanics)

A sealed capsule from the observatory's founder is finally opened. Inside is a note describing a "courier star" placed in an elliptical orbit around a compact object of mass $1.50M_\odot$. The orbit has semi-major axis 4.00 AU and eccentricity 0.60. The note says the courier carries the next piece of the archive and will be most vulnerable when it dives through periapsis. Rayan is asked to determine the speed at that instant.
Deduce the courier's orbital speed at periapsis in $\mathrm{km\,s^{-1}}$.

SOLUTION

The periapsis distance is

$$r_p = a(1-e)$$

$$r_p = 4.00(1-0.60) = 1.60\,\mathrm{AU}$$

The vis-viva equation gives the speed at any point of the ellipse.

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

Here

$$M = 1.50(1.989\times10^{30}) = 2.9835\times10^{30}\,\mathrm{kg}$$

$$r_p = 1.60(1.496\times10^{11}) = 2.3936\times10^{11}\,\mathrm{m}$$

$$a = 4.00(1.496\times10^{11}) = 5.984\times10^{11}\,\mathrm{m}$$

$$v_p = \sqrt{(6.674 \times 10^{-11})(2.9835 \times 10^{30}) \left(\frac{2}{2.3936 \times 10^{11}} - \frac{1}{5.984 \times 10^{11}} \right)}$$

$$v_p \approx 3.648 \times 10^4 \,\mathrm{m\,s^{-1}}$$

$$\boxed{v_p \approx 36.5 \,\mathrm{km\,s^{-1}}}$$

Question 4: The Star Behind the Glass (Astrophysics)

When Mara returns to the chamber of black glass, the star behind the narrow opening has changed. Before the disturbance, its thermal spectrum reached its maximum at 500 nm. After the disturbance, the maximum is found at 400 nm. At the same time, the bolometric flux measured at Earth has become 6.25 times larger. The star is sufficiently distant that its distance can be treated as unchanged during the event.

The archive gives only the blackbody relation between emitted power, surface area, and effective temperature, together with the two observations. Rayan suspects that the star did not simply become hotter: part of the change came from its size.

What is the ratio of the new radius to the original radius?

SOLUTION

For thermal radiation from a spherical blackbody, the emitted luminosity is proportional to its surface area and to the fourth power of its effective temperature. We first extract the temperature change from the movement of the spectral peak.

The peak wavelength of a blackbody varies inversely with temperature, so

$$\frac{T_2}{T_1} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{T_2}{T_1} = \frac{500}{400} = 1.25$$

At fixed distance, the observed bolometric flux changes in the same proportion as the luminosity.

$$\frac{F_2}{F_1} = \left(\frac{R_2}{R_1}\right)^2 \left(\frac{T_2}{T_1}\right)^4$$

Insert the measured flux ratio.

$$6.25 = \left(\frac{R_2}{R_1}\right)^2 (1.25)^4$$

$$\left(\frac{R_2}{R_1}\right)^2 = \frac{6.25}{2.44140625} = 2.56$$

$$\boxed{\frac{R_2}{R_1} = 1.60}$$

Question 5: The World That Fell Through (Planetology)

The chamber floor opens beneath Rayan, revealing a small world suspended over a dark gravitational well. Its inscription calls it the "Fallen World." The only direct measurements written on the wall are a mass of 4.80×10^{24} kg and a mean density of $6000\,\mathrm{kg\,m^{-3}}$.
A second instrument gives the gravitational acceleration at a point 300 km above the surface as $6.50\,\mathrm{m\,s^{-2}}$. Rayan initially assumes this is enough to determine the surface gravity directly. Mara notices a warning carved into the stone: "The height is measured from the skin, not from the heart." The world is spherical, the outside field is Newtonian, and its mass is concentrated as if all of it were at its center.
Uncover the surface gravitational acceleration of the Fallen World in $\mathrm{m\,s^{-2}}$.

SOLUTION

The mass and density determine the physical size of the world, while the measurement above the surface provides an independent check. The useful starting point is the definition of density.

$$\rho = \frac{M}{V}$$

For a sphere,

$$V = \frac{4}{3}\pi R^3$$

Combining the two,

$$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$

$$R = \left(\frac{3(4.80 \times 10^{24})}{4\pi(6000)}\right)^{1/3}$$

$$R \approx 5.759 \times 10^6\ \mathrm{m}$$

Now use the fact that outside a spherical body the gravitational acceleration depends on the distance from its center. At the point measured by the instrument,

$$r = R + 300\,\mathrm{km}$$

$$r = 5.759 \times 10^6 + 3.00 \times 10^5$$

$$r = 6.059 \times 10^6\ \mathrm{m}$$

The gravitational field there is

$$g(r) = \frac{GM}{r^2}$$

The quantity we need is the field at the surface, not at the instrument, so

$$g(R) = \frac{GM}{R^2}$$

Using the radius obtained from the density,

$$g(R) = \frac{(6.674 \times 10^{-11})(4.80 \times 10^{24})}{(5.759 \times 10^6)^2}$$

$$\boxed{g(R) \approx 9.66\,\mathrm{m\,s^{-2}}}$$

The $6.50\,\mathrm{m\,s^{-2}}$ measurement is consistent with a higher altitude, but it is not the surface value. The trap is treating the stated 300 km as the distance from the center.

Question 6: The Red Thread (Obs Ast)

The archive contains a red line drawn across several spectral plates. Dr. Leila realizes that it is not ink but a shifted hydrogen line. The laboratory reference wavelength is 656.28 nm, while the distant source is observed at 657.15 nm. Rayan is told to treat the motion as non-relativistic. The red line is the only surviving clue to how quickly the source is escaping the station.
Read off the source's radial recession speed in $\mathrm{km\,s^{-1}}$.

SOLUTION

For a non-relativistic redshift

$$z = \frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$$

The wavelength shift is

$$\Delta\lambda = 657.15 - 656.28 = 0.87\,\mathrm{nm}$$

Thus

$$v = c\frac{\Delta\lambda}{\lambda_0}$$

$$v = (3.00 \times 10^5)\frac{0.87}{656.28}$$

$$\boxed{v \approx 3.98 \times 10^2\,\mathrm{km\,s^{-1}}}$$

Question 7: The Ledger of Two Suns (Astrophysics)

The black-glass archive contains a page from an old observatory log. It describes two stars that repeatedly pass in front of one another. The astronomer recorded a relative orbital semi-major axis of 0.240 AU and a period of 18.0 days. No mass is written anywhere on the page.
Rayan remembers only the geometric definition of orbital speed for uniform circular motion and Newton's inverse-square gravitational force. The binary is sufficiently wide that the two stars may be treated with Newtonian gravity, and the measured semi-major axis refers to their relative orbit.
Recover the total mass of the binary system in solar masses.

SOLUTION

The two stars orbit their common center of mass, but the relative orbit can be treated as the motion of one body with respect to the other. Let the total mass be M and the relative separation be r.

For the relative circular motion, the required centripetal acceleration is

$$a = \frac{v^2}{r}$$

The relative orbital speed is the circumference covered in one period divided by that period.

$$v = \frac{2\pi r}{P}$$

Newtonian gravity gives the acceleration scale

$$a = \frac{GM}{r^2}$$

Set the two descriptions of the same acceleration equal.

$$\frac{v^2}{r} = \frac{GM}{r^2}$$

Substitute the orbital speed.

$$\frac{1}{r}\left(\frac{2\pi r}{P}\right)^2 = \frac{GM}{r^2}$$

After simplifying,

$$M = \frac{4\pi^2 r^3}{GP^2}$$

Convert the measurements.

$$r = 0.240(1.496 \times 10^{11}) = 3.5904 \times 10^{10}\,\mathrm{m}$$

$$P = 18.0(86400) = 1.5552 \times 10^{6}\,\mathrm{s}$$

Then

$$M = \frac{4\pi^2(3.5904 \times 10^{10})^3}{(6.674 \times 10^{-11})(1.5552 \times 10^6)^2}$$

$$M \approx 1.132 \times 10^{31}\,\mathrm{kg}$$

Divide by the solar mass.

$$\frac{M}{M_\odot} = \frac{1.132 \times 10^{31}}{1.989 \times 10^{30}}$$

$$\boxed{M \approx 5.69\,M_\odot}$$

Question 8: The Radius of the Invisible Furnace (Astrophysics)

The observatory's mechanical lock responds to a dark circular region in an old interferometric image. The archive gives no mass. It records an angular diameter of 9.60×10^{-16} radians for the dark region and places the object at a distance of 2.00 kpc.
The inscription adds only one statement: for a non-rotating black hole, the boundary of no return occurs where the escape speed reaches the speed of light. Rayan must recover the missing mass from the image.
Reconstruct the mass of the black hole in solar masses.

SOLUTION

At the boundary of no return, the speed required to escape equals the speed of light. Start from the Newtonian escape-speed relation, obtained by setting the kinetic energy equal to the gravitational binding energy.

$$\frac{1}{2}mv_{\text{esc}}^2 = \frac{GMm}{r}$$

Cancel the test mass and set $v_{\text{esc}} = c$ at the critical radius.

$$\frac{1}{2}c^2 = \frac{GM}{r_s}$$

Rearranging gives the radius of the boundary.

$$r_s = \frac{2GM}{c^2}$$

The image provides the angular diameter, so the small-angle relation gives the physical diameter.

$$D = \theta d$$

$$d = 2.00(10^3)(3.086 \times 10^{16}) = 6.172 \times 10^{19}\,\text{m}$$

$$D = (9.60 \times 10^{-16})(6.172 \times 10^{19})$$

$$D \approx 5.925 \times 10^4\,\text{m}$$

Thus

$$r_s = \frac{D}{2} \approx 2.962 \times 10^4\,\text{m}$$

Now solve the escape-speed relation for the mass.

$$M = \frac{r_s c^2}{2G}$$

$$M = \frac{(2.962 \times 10^4)(3.00 \times 10^8)^2}{2(6.674 \times 10^{-11})}$$

$$M \approx 2.00 \times 10^{31}\,\text{kg}$$

$$\frac{M}{M_\odot} = \frac{2.00 \times 10^{31}}{1.989 \times 10^{30}}$$

$$\boxed{M \approx 10.1\, M_\odot}$$

Question 9: The Age Written in Red (Cosmology)

At midnight, the archive displays one final observation. A distant galaxy is seen with a spectral line whose wavelength is 2.50 times the wavelength measured in the laboratory. The observatory's cosmology panel uses a matter-dominated model. The strange part is that the panel has stopped displaying distance and instead asks for the age of the universe when the light began its journey. How old was the universe, in billions of years, when the light began its journey?

SOLUTION

The redshift relation gives the scale factor at emission.

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = 2.50$$

$$a_{\text{em}} = \frac{1}{1+z} = \frac{1}{2.50} = 0.400$$

For a matter-dominated universe

$$a = \left(\frac{t}{t_0}\right)^{2/3}$$

Thus

$$t = t_0 a^{3/2}$$

$$t = 13.8(0.400)^{3/2}$$

$$t \approx 13.8(0.25298)$$

$$\boxed{t \approx 3.49 \text{ billion years}}$$

Question 10: The Clock with Two Histories (Cosmology)

The tenth chamber does not contain an instrument. It contains a clock. Its face is divided into two cosmic eras. According to the inscription, the scale factor of the simulated universe obeyed radiation-like expansion with $a \propto t^{1/2}$ until $t = 1.00$ billion years, and then changed to matter-like expansion with $a \propto t^{2/3}$. The simulation is normalized so that $a(13.8\,\text{Gyr}) = 1$ and $a(1.00\,\text{Gyr}) = 0.20$. A signal now arrives with wavelength seven times its emitted wavelength. Rayan must identify which side of the clock the signal came from.

Pin down the cosmic age, in billions of years, at which the signal was emitted.

SOLUTION

The observed wavelength gives the emission scale factor.

$$1 + z = 7$$

$$a_{\mathrm{em}} = \frac{1}{7} = 0.142857$$

The transition occurred at $a_{\mathrm{tr}} = 0.20$, so the emission happened before the transition. For the earlier radiation-like era

$$a \propto t^{1/2}$$

Using the normalization at $t_{\mathrm{tr}} = 1.00\,\mathrm{Gyr}$

$$\frac{a_{\mathrm{em}}}{a_{\mathrm{tr}}} = \left(\frac{t_{\mathrm{em}}}{t_{\mathrm{tr}}}\right)^{1/2}$$

$$\frac{1/7}{0.20} = \left(\frac{t_{\mathrm{em}}}{1.00\,\mathrm{Gyr}}\right)^{1/2}$$

$$\frac{5}{7} = \left(\frac{t_{\mathrm{em}}}{1.00\,\mathrm{Gyr}}\right)^{1/2}$$

$$t_{\mathrm{em}} = \left(\frac{5}{7}\right)^{2} (1.00\,\mathrm{Gyr})$$

$$\boxed{t_{\mathrm{em}} = 0.510\,\mathrm{Gyr}}$$