GLOBAL ASTRONOMY & ASTROPHYSICS CHALLENGE

ROUND 2

Knockout Bank: Grand Final

10 Free-Response Questions with Full Worked Solutions

TOPICS COVERED

Observational Astronomy · Orbital Mechanics · Astrophysics
Planetology · Compact Objects · Cosmology

2026

Reference Sheet

Physical and Astronomical Constants

Quantity	Symbol	Value
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Solar mass	$M_\odot$	$1.989 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_\odot$	$6.957 \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_\odot$	$3.828 \times 10^{26}\,\mathrm{W}$
Solar effective temperature	$T_\odot$	$5772\,\mathrm{K}$
Astronomical unit	AU	$1.496 \times 10^{11}\,\mathrm{m}$
Parsec	pc	$3.086 \times 10^{16}\,\mathrm{m}$
Stefan–Boltzmann constant	σ	$5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Wien's displacement constant	b	$2.898 \times 10^{-3}\,\mathrm{m\,K}$
Reduced Planck constant	$\hbar$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Electron mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg}$
Earth mass	$M_\oplus$	$5.972 \times 10^{24}\,\mathrm{kg}$
Earth radius	$R_\oplus$	$6.371 \times 10^{6}\,\mathrm{m}$
Hubble constant	H_0	$70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$
Megaparsec	1 Mpc	$3.086 \times 10^{19}\,\mathrm{km}$
Arcseconds per radian		206 265
CMB temperature today	T_0	$2.725\,\mathrm{K}$
Sun main-sequence lifetime		10 Gyr
Age of the universe today	t_0	13.8 Gyr
Year	1 yr	$3.156 \times 10^{7}\,\mathrm{s}$
Thomson cross-section	σ_T	$6.65 \times 10^{-29}\,\mathrm{m^2}$
Proton mass	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$

Useful Relations

Diffraction Resolution

$$\theta = \frac{1.22\,\lambda}{D}$$

Force on a Moving Charge

$$F = q\,v\,B$$

Circular-Orbit Binding

$$E = -\frac{GMm}{2a}$$

Wien Displacement

$$\lambda_{\max} T = b$$

Self-Gravity Binding

$$E_{\text{bind}} = \frac{3GM^2}{5R}$$

Cosmic Dilution

$$1 + z = \frac{1}{a}, \quad \rho_m \propto a^{-3}, \quad \rho_r \propto a^{-4}, \quad T \propto a^{-1}$$

These six master equations seed every derivation in this booklet. Each question asks for a specialised quantity; obtaining it from them always requires a short chain of reasoning. None of the answers can be reached by substituting numbers into the master equations directly.

Questions

Stage 3: The Finals

The last transmission: a voyage through compact stars, wandering worlds, and the edge of the observable universe.

Question 21: The Needle in the Radio Sky (Obs Ast)

Years later, Rayan receives the last transmission from an automated radio observatory. The transmitter reports two compact sources separated by only 0.030 arcseconds. The observatory works at a wavelength of 1.30 cm and uses an interferometric baseline. The old engineer's note says the sources are "as far apart as a needle's width appears from Cairo to the Moon."
How long must the baseline be, in kilometres, to barely resolve the two sources?

SOLUTION

Convert the angular separation to radians.

$$\theta = 0.030\left(\frac{\pi}{180 \times 3600}\right) \approx 1.454 \times 10^{-7}\,\mathrm{rad}$$

The wavelength is

$$\lambda = 1.30\,\mathrm{cm} = 1.30 \times 10^{-2}\,\mathrm{m}$$

Using the Rayleigh criterion

$$B = \frac{1.22\lambda}{\theta}$$

$$B = \frac{1.22(1.30 \times 10^{-2})}{1.454 \times 10^{-7}}$$

$$B \approx 1.091 \times 10^{5}\,\mathrm{m}$$

$$\boxed{B \approx 109\,\mathrm{km}}$$

Question 22: The Orbit beneath the Iron Star (Obs Ast: Orbital Mechanics)

A message from the radio observatory points to a compact star of mass $1.40M_\odot$. A probe circles it on an orbit with semi-major axis 1.20×10^9 m and eccentricity 0.20. Each revolution rotates the line of apsides by a tiny amount, and the message claims that general relativity is responsible. The orbit is far enough outside the compact object's strongest-field region for the leading-order precession formula to be used.
Evaluate the periapsis advance per orbit in radians, using $\Delta\omega = 6\pi GM/[c^2 a(1-e^2)]$.

SOLUTION

The mass is

$$M = 1.40(1.989 \times 10^{30}) = 2.7846 \times 10^{30}\,\text{kg}$$

The eccentricity factor is

$$1 - e^2 = 1 - 0.20^2 = 0.96$$

Substitute into the relativistic precession formula.

$$\Delta\omega = \frac{6\pi(6.674 \times 10^{-11})(2.7846 \times 10^{30})}{(3.00 \times 10^8)^2(1.20 \times 10^9)(0.96)}$$

$$\Delta\omega \approx 3.379 \times 10^{-5}\,\text{rad}$$

$$\boxed{\Delta\omega \approx 3.38 \times 10^{-5}\,\text{rad per orbit}}$$

Question 23: The One-Second Shadow (Obs Ast)

Two compact radio sources in a distant galaxy are separated by 0.12 pc. The galaxy is 75 Mpc away. Rayan's interferometer has a maximum baseline of 1.20×10^5 m.
The array team wants to know the longest wavelength that can still distinguish the two sources. Rayan is told only that the angular width of the diffraction pattern decreases as the baseline becomes longer and that the Rayleigh criterion corresponds to a dimensionless numerical factor of 1.22 multiplying the ratio of observing wavelength to baseline.
Identify the longest usable wavelength in centimetres.

SOLUTION

The physical separation and distance give the angular separation.

$$\theta = \frac{s}{D}$$

Because both quantities can be expressed in parsecs,

$$\theta = \frac{0.12}{75 \times 10^6}$$

$$\theta = 1.60 \times 10^{-9}\,\text{rad}$$

At the Rayleigh limit, the angular resolution is 1.22 times the wavelength divided by the baseline. Equating the resolution to the actual separation gives

$$\theta = 1.22\frac{\lambda}{B}$$

Solve for the largest wavelength the array can tolerate.

$$\lambda = \frac{\theta B}{1.22}$$

$$\lambda = \frac{(1.60 \times 10^{-9})(1.20 \times 10^{5})}{1.22}$$

$$\lambda \approx 1.574 \times 10^{-4}\,\mathrm{m}$$

$$\boxed{\lambda \approx 0.0157\,\mathrm{cm}}$$

Question 24: The Neutron Star's Lost Century (Astrophysics)

The message from the radio observatory contains an image of a neutron star being born. Matter of mean density $4.0 \times 10^{17}\,\mathrm{kg\,m^{-3}}$ collapses to a sphere, and the total mass is $1.40M_{\odot}$. The collapse releases the gravitational self-energy of a uniform sphere, and the archive says that 99% of this released energy leaves in neutrinos. Rayan must identify the energy carried away in that invisible burst.
Track the neutrino energy in joules.

SOLUTION

The collapsed star is a uniform sphere of density ρ. Its radius follows from mass and density.

$$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$

$$M = 1.40(1.989 \times 10^{30}) = 2.7846 \times 10^{30}\,\mathrm{kg}$$

$$R = \left(\frac{3(2.7846 \times 10^{30})}{4\pi(4.0 \times 10^{17})}\right)^{1/3}$$

$$R \approx 1.186 \times 10^{4}\,\mathrm{m}$$

The released energy is the gravitational binding energy of a uniform sphere.

$$E_{\mathrm{bind}} = \frac{3GM^2}{5R}$$

$$E_{\mathrm{bind}} = \frac{3(6.674 \times 10^{-11})(2.7846 \times 10^{30})^2}{5(1.186 \times 10^{4})}$$

$$E_{\mathrm{bind}} \approx 2.62 \times 10^{46}\,\mathrm{J}$$

The neutrino energy is

$$E_{\nu} = 0.99E_{\mathrm{bind}}$$

$$\boxed{E_{\nu} \approx 2.59 \times 10^{46}\,\mathrm{J}}$$

Question 25: The Black Hole Candle (Astrophysics)

Inside the transmitter's maintenance log, Mara finds a record of a faint source believed to be a primordial black hole. The source lies at redshift $z = 0.50$, and its observed thermal spectrum peaks at a wavelength of 7.90×10^{-6} m. The archive supplies the Hawking temperature law for a non-rotating black hole,

$$T_H = \frac{\hbar c^3}{8\pi G k_B M},$$

and the constants needed to evaluate it. Use Wien's displacement law for the thermal spectrum together with the Hawking law to determine the mass of the distant black hole.
Retrieve the mass of the black hole in kilograms.

SOLUTION

The wavelength recorded by the telescope has been stretched by cosmic expansion. The emitted wavelength is related to the observed wavelength by

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}}$$

Thus

$$\lambda_{\text{em}} = \frac{7.90 \times 10^{-6}}{1.50}$$

$$\lambda_{\text{em}} = 5.27 \times 10^{-6}\,\text{m}$$

Wien's displacement law connects the emitted peak wavelength with the temperature.

$$\lambda_{\text{em}} T = b$$

$$T = \frac{2.898 \times 10^{-3}}{5.27 \times 10^{-6}}$$

$$T \approx 5.50 \times 10^{2}\,\text{K}$$

Rearranging the Hawking temperature law gives the mass.

$$M = \frac{\hbar c^3}{8\pi G k_B T}$$

$$M = \frac{(1.055 \times 10^{-34})(3.00 \times 10^{8})^3}{8\pi(6.674 \times 10^{-11})(1.381 \times 10^{-23})(5.50 \times 10^{2})}$$

$$M \approx 2.24 \times 10^{20}\,\text{kg}$$

$$\boxed{M \approx 2.24 \times 10^{20}\,\text{kg}}$$

Question 26: The Radio Ghost of Cygnus (Astrophysics)

Rayan and Mara turn the array toward a faint supernova remnant whose expanding shell is several light-years across. Polarization maps reveal a magnetic field of 2.0×10^{-5} T threaded through the shocked plasma. The remnant shines by synchrotron radiation, and its optical spectrum rises to a characteristic frequency of 4.2×10^{14} Hz.
The archive contains no electron energies. Instead, it records the physical origin of the motion: an electron spirals around the magnetic field, with the magnetic force providing the centripetal force. In the ultra-relativistic regime, the inertia term contains γm_e, and the observed synchrotron frequency is enhanced by the relativistic beaming of the radiation. Begin from these physical statements rather than a ready-made synchrotron formula.
Obtain the characteristic energy of the emitting electrons in teraelectronvolts (TeV).

SOLUTION

The magnetic force provides the centripetal force for an electron moving perpendicular to the field.

$$evB = \frac{\gamma m_e v^2}{r}$$

Cancel one factor of v.

$$\frac{v}{r} = \frac{eB}{\gamma m_e}$$

The relativistic gyration frequency is

$$\nu_g = \frac{1}{2\pi}\frac{v}{r}$$

Substitute the expression obtained from the force balance.

$$\nu_g = \frac{eB}{2\pi\gamma m_e}$$

The relativistic beaming of synchrotron radiation makes the characteristic observed frequency scale as $\gamma^3\nu_g$. For the usual pitch-angle choice used in this model, the characteristic frequency is

$$\nu_c = \frac{3}{2}\gamma^3\nu_g$$

Substituting the gyration frequency,

$$\nu_c = \frac{3eB}{4\pi m_e}\gamma^2$$

Solve for the Lorentz factor.

$$\gamma = \sqrt{\frac{4\pi m_e \nu_c}{3eB}}$$

Insert the observed synchrotron frequency and magnetic field.

$$\gamma = \sqrt{\frac{4\pi(9.11 \times 10^{-31})(4.2 \times 10^{14})}{3(1.602 \times 10^{-19})(2.0 \times 10^{-5})}}$$

$$\gamma \approx 2.24 \times 10^4$$

For an ultra-relativistic electron, the energy is approximately

$$E \approx \gamma m_e c^2$$

$$E = (2.24 \times 10^4)(9.11 \times 10^{-31})(3.00 \times 10^8)^2$$

$$E \approx 1.84 \times 10^{-9}\,\text{J}$$

Using $1\,\text{TeV} = 1.602 \times 10^{-7}\,\text{J}$,

$$E \approx \frac{1.84 \times 10^{-9}}{1.602 \times 10^{-7}}\,\text{TeV}$$

$$\boxed{E \approx 1.15 \times 10^{-2}\,\text{TeV}}$$

Question 27: The White Dwarf's Broken Rule (Astrophysics)

Near the end of the transmission, the archive shows a white dwarf approaching the limit at which electron degeneracy can no longer support it. The original star is carbon-rich, so its electron fraction is $Y_e = 0.50$. A hypothetical reaction changes the average composition and reduces the electron fraction to 0.46.

The archive gives the ultra-relativistic equation of state in the form

$$P = K\rho^{4/3}$$

and notes that the mass of the limiting configuration depends on the constant K as $M \propto K^{3/2}$. The relation between K and the electron fraction follows from the degeneracy pressure of relativistic electrons.

Settle the new maximum supported mass in solar masses if the original limit was $1.456 M_\odot$.

SOLUTION

For relativistic electrons, the degeneracy pressure scales with the fourth power of the Fermi momentum, while the Fermi momentum scales with the number density of electrons to the one-third power. Because the electron number density is proportional to $Y_e\rho$, the coefficient of the relativistic equation of state scales as

$$K \propto Y_e^{4/3}$$

The limiting mass scales as

$$M_{\max} \propto K^{3/2}$$

Combining the two proportionalities gives

$$M_{\max} \propto \left(Y_e^{4/3}\right)^{3/2}$$

$$M_{\max} \propto Y_e^2$$

Thus the ratio of the new and old limits is

$$\frac{M_2}{M_1} = \left(\frac{Y_{e,2}}{Y_{e,1}}\right)^2$$

$$M_2 = 1.456\left(\frac{0.46}{0.50}\right)^2 M_\odot$$

$$M_2 = 1.456(0.8464)M_\odot$$

$$\boxed{M_2 \approx 1.23M_\odot}$$

Question 28: The Planet that Burned Its Path (Planetology)

The final planet in the archive is not a world seen in a telescope but a simulation of a captured Earth-mass planet. It moves from a circular orbit at 5.00 AU to another circular orbit at 1.00 AU around a Sun-mass star. The simulation assumes that the planet loses its orbital energy to a surrounding disk, with no other energy sink. The planet has exactly the mass of the Earth, which the archive's table of constants supplies. Rayan must determine how much orbital energy the disk receives.
Account for the orbital energy dissipated in the disk in joules.

SOLUTION

For a circular orbit, the orbital energy is

$$E = -\frac{GMm}{2a}$$

The dissipated amount is the decrease in orbital energy magnitude.

$$\Delta E = \frac{GMm}{2}\left(\frac{1}{a_f} - \frac{1}{a_i}\right)$$

Use

$$M = 1.989 \times 10^{30}\,\text{kg}, \qquad m = 5.972 \times 10^{24}\,\text{kg}$$

$$a_i = 5.00(1.496 \times 10^{11}) = 7.48 \times 10^{11}\,\text{m}$$

$$a_f = 1.00(1.496 \times 10^{11}) = 1.496 \times 10^{11}\,\text{m}$$

Then

$$\Delta E \approx 2.11 \times 10^{33}\,\text{J}$$

$$\boxed{\Delta E \approx 2.11 \times 10^{33}\,\text{J}}$$

Question 29: The Universe in the Mirror (Cosmology)

At last, the transmitter becomes a mirror and shows the universe at the moment when matter and radiation had exactly equal energy densities. The archive gives today's density parameters as

$$\Omega_r = 9.0 \times 10^{-5}, \qquad \Omega_m = 0.30.$$

The mirror gives no redshift and no scale factor. It only records that radiation density is controlled by the volume dilution of photons together with the redshift of their energies, while non-relativistic matter density changes only because of volume dilution.
Travel back to equality: what was the CMB temperature in kelvin?

SOLUTION

The physical volume of the universe scales as a^3. Radiation undergoes an additional energy loss because each photon is redshifted, so its energy density gains one more factor of a^{-1}. Matter does not have this extra redshift factor.
Thus the density scalings are

$$\rho_r = \rho_{r0} a^{-4}$$

and

$$\rho_m = \rho_{m0} a^{-3}$$

At equality,

$$\rho_r = \rho_m$$

Using the present-day density parameters as proportional measures of the present densities,

$$\Omega_r a^{-4} = \Omega_m a^{-3}$$

Cancel the common factor a^{-3}.

$$\Omega_r a^{-1} = \Omega_m$$

Therefore

$$a = \frac{\Omega_r}{\Omega_m}$$

$$a = \frac{9.0 \times 10^{-5}}{0.30} = 3.00 \times 10^{-4}$$

The wavelength of every freely propagating photon stretches in direct proportion to the scale factor. Because photon energy is inversely proportional to wavelength, the radiation temperature follows the inverse scale factor.

$$T = T_0 \frac{1}{a}$$

$$T = 2.725 \times \frac{1}{3.00 \times 10^{-4}}$$

$$\boxed{T \approx 9.08 \times 10^3\,\mathrm{K}}$$

Question 30: The Last Horizon (Cosmology)

The last transmission contains no image, only an instruction. The archive describes a flat universe with radiation, matter, and a cosmological constant:

$$\Omega_r = 0.10, \qquad \Omega_m = 0.30, \qquad \Omega_\Lambda = 0.60.$$

Rayan is told to calculate the proper particle-horizon distance today. The archive provides

$$d_{\mathrm{p}}(t_0) = \frac{c}{H_0} \int_0^1 \frac{da}{a^2 \sqrt{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda}}.$$

The machine has enough precision to evaluate the dimensionless integral numerically. The twist is that the integral is taken all the way back to $a = 0$, where radiation dominates the early universe, while matter and the cosmological constant control the later evolution.
Express the present particle-horizon distance as a multiple of c/H_0 and evaluate the numerical coefficient.

SOLUTION

Insert the three density parameters into the integral.

$$d_{\mathrm{p}} = \frac{c}{H_0} \int_0^1 \frac{da}{a^2 \sqrt{0.10 a^{-4} + 0.30 a^{-3} + 0.60}}$$

The dimensionless factor is

$$I = \int_0^1 \frac{da}{a^2 \sqrt{0.10 a^{-4} + 0.30 a^{-3} + 0.60}}$$

Numerical evaluation gives

$$I \approx 1.912$$

The required multiple is

$$\boxed{d_{\mathrm{p}} \approx 1.912 \, \frac{c}{H_0}}$$